APPENDIX B

ENGINEERING PRINCIPLES OF SHEAR WALLS WITH SLOPING TOP PLATES

APA—The Engineered Wood Association and others have tested only *rectangular* shear wall assemblies. Most of these have been eight feet high and eight feet wide, with more recent tests also involving four-foot wide walls. What happens when the top and bottom members of the shear wall are not parallel? We can no longer divide the wall into square shear elements. If we have a fully sheathed gable end wall, then it may be safe to assume that the symmetry of the wall will carry diaphragm forces to the rectangular portion of the shear wall below. Many times, though, the shear wall or shear wall segment only slopes one direction, as shown in Figure 1.

Figure 1 Shear wall segments with mono-sloped top plates

Designers sometimes "simplify" the overturning and tie-down force calculations for a sloped wall by assuming that the total force on the wall acts horizontally at the average wall height. This simplification is incorrect, and can lead to seriously underestimated tie-down forces. An analysis of a mono-sloped shear wall follows.

By definition, the roof diaphragm only carries forces within the plane of the sheathing. Therefore the force delivered from the diaphragm acts along the sloping top member of the shear wall. Figure 2 shows a free-body diagram of a shear wall with a sloping top. The figure illustrates how the diaphragm force creates a greater overturning moment (OTM) about the base of the taller end-post than it does about the base of the shorter end-post. Since the same distance between the end-posts is used to find the tie-down force at either end of the wall, we find that the tie-down force at the short end-post is always greater than that at the tall end-post. (Note that in this discussion we will focus on the tie-down forces; the compressive

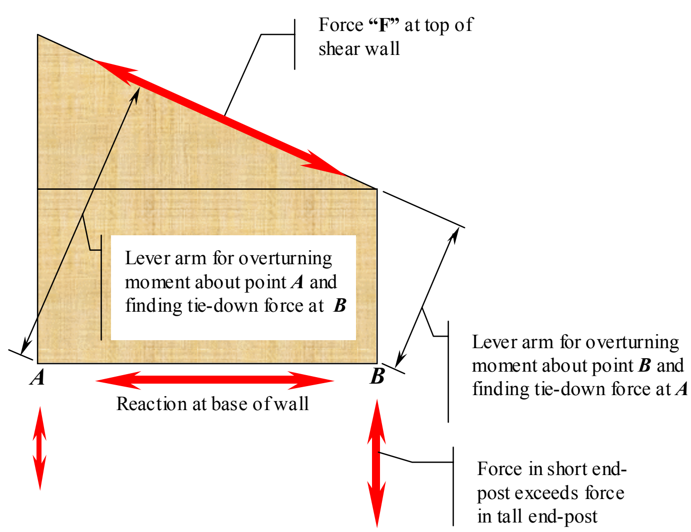

Figure 2 Schematic Free-body Diagram of shear wall with sloped top plate

forces in the end-posts will have the same relationship in the case where the load at the top of the wall reverses.)

Figure 3 shows an example of a sloping shear wall with actual dimensions and forces.

Finding the OTM about point **A**, we get:

$$(\text{OTM})_A = 12.93 \text{ feet } (2{,}500 \text{ pounds}) = 32{,}325 \text{ foot-pounds}$$

From this we can determine the tie-down force, **T**, at the short end-post (at Point **B**) as:

$$\mathbf{T}_B = 32{,}325 \text{ foot-pounds}/12 \text{ feet} = 2{,}694 \text{ pounds}$$

The OTM about point **B** is:

$$(\text{OTM})_B = 6.93 \text{ feet } (2{,}500\text{pounds}) = 17{,}325 \text{ foot-pounds}$$

Which gives the tie-down force at the tall end-post (Point **A**) as:

$$\mathbf{T}_A = 17{,}325 \text{ foot-pounds}/12 \text{ feet} = 1{,}444 \text{ pounds}$$

In this example the tie-down force at the short end-post is almost twice that at the tall end-post.

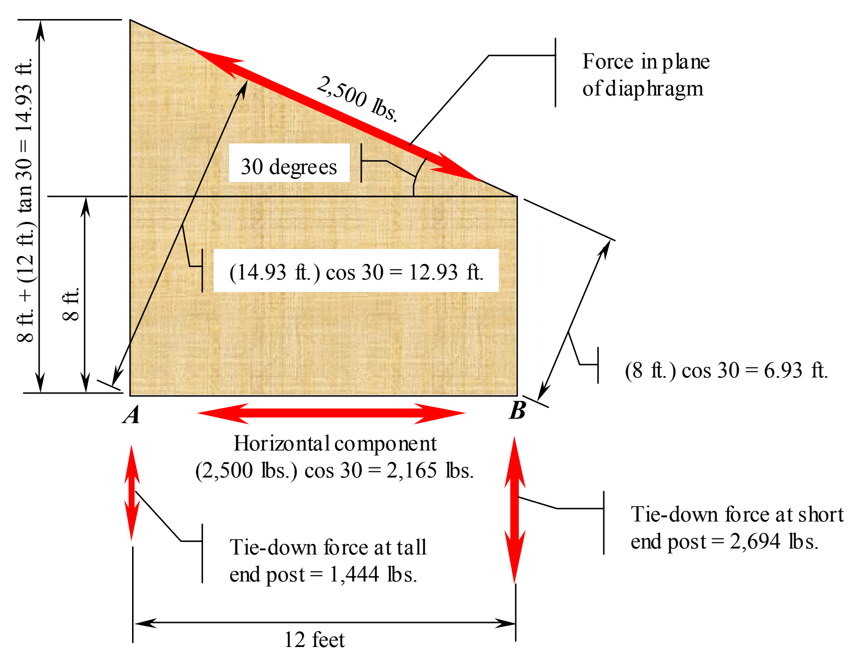

Figure 3 Example wall illustrating different tie-down forces at each end-post

(If we assumed, incorrectly, that the diaphragm force acted horizontally at the average wall height, we would get the same tie-down force at each end of the wall of (11.47 feet) (2,500 pounds) (cos 30)/12 feet = 2,068 pounds)—this is the *average* of the correct tie-down forces.)

If we break the sloped wall into two segments we can analyze them separately and better understand how the wall works as a whole. We have established that the diaphragm force acts parallel to the sloping top of the triangular wall segment. The reaction from the main (rectangular) portion of the wall acts horizontally on the bottom of the triangular segment. By the principles of statics, the triangular wall segment is a "three-force body." The third force must act through a point concurrent with the first two. This means that a vertical force must act at the intersection point of the sloping top plate and the horizontal wall plate. Figure 4 illustrates the triangular portion of the shear wall. Essentially the triangular segment of shear panels becomes a huge gusset that connects the sloping and horizontal members. For the example in Figure 3, the forces would be as follows:

Horizontal reaction = (2,500 pounds) cos 30 = 2,165 pounds

Vertical reaction = (2,500 pounds) sin 30 = 1,250 pounds

This indicates that we must provide a tie-down force of 1,250 pounds *to the sloped member of the triangular section of the shear wall.* One way to accomplish this is by installing a strap from the short end-post up and over the sloping top member of the shear wall, as shown in the Wood-frame Shear Wall Construction Guide. We must remember that the "additional" vertical force component acts at the sloped top member of the shear wall—simply increasing the tie-down connection to the base of the short end-post will allow the top portion of the wall to lift up and away.

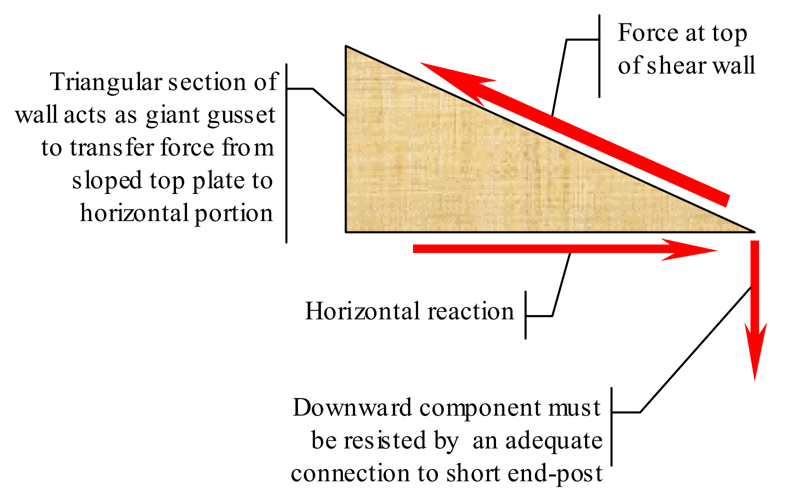

Figure 4 Free-body Diagram of triangular portion of wall. Three forces must act through the point at the top of the short end of the wall.

If we consider only the rectangular portion of the wall, we can then add the vertical reaction from the triangular portion to find the overall tie-down forces. Using the shear wall from the previous example, the horizontal component acting at the top of the rectangular portion of the wall would require a tie-down force of (2,165 pounds) 8 feet/12 feet = 1,444 pounds. This is the same force we found for the tall end-post as determined in Figure 3. If we add the 1,250 lb. vertical

component shown in Figure 4 to 1,444 lbs, we get 2,694 pounds. This is the same force determined in Figure 3 as the tie-down force at the short end-post.

The preceding gives us another method to find the tie-down force at the short end-post: The horizontal component of the diaphragm force (typically determined in a lateral analysis) multiplied by the tangent of the roof angle gives the vertical force required to tie the triangular wall segment to the short end-post. Figure 5 illustrates how we can add the forces determined in the "standard" rectangular wall and the triangular segment to get the overall forces on the wall.

Note that this is a ***theoretical*** analysis, and does not account for the deformation of the wall segments, added strength from sloped rafters that may be present at the top of the wall, bending in the nails, panel buckling and so forth. Full scale testing of mono-sloped wall segments could verify or disprove the assumptions stated above. Until such testing is undertaken, this author recommends the preceding approach. Once you get used to the process, it is just as easy as the more common approach of finding the overturning moment due to a horizontal load applied at the average wall height: calculating the vertical component of force in the short end post involves no more effort than finding the average wall height.

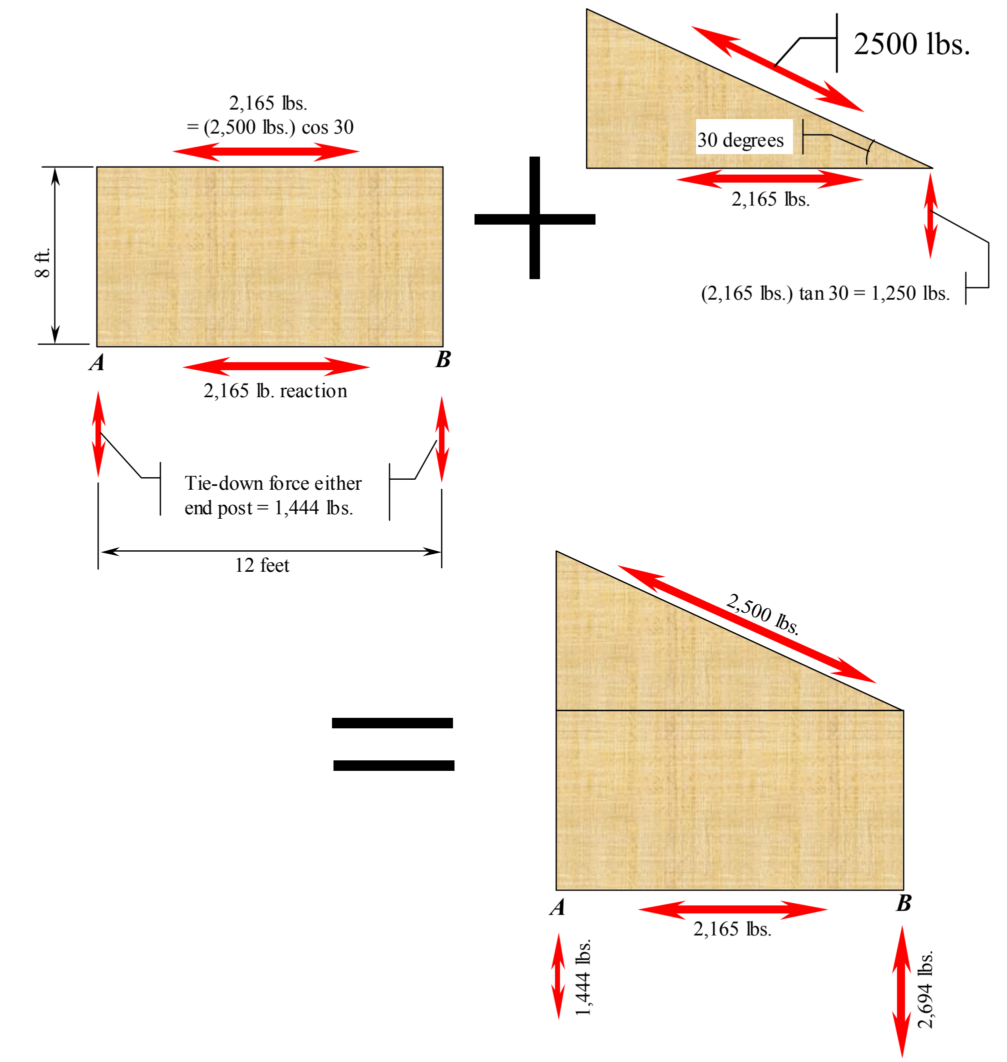

Figure 5 Example showing "super-position" of forces

APPENDIX TO THE APPENDIX

DISCUSSION OF DIAPHRAGM FORCES

We must believe in Statics. I would love to see any other analysis of a shear wall with a sloped top chord, as long as the analysis follows the principles of statics. (That includes how the load gets to the top of the shear wall.)

Some engineers have proposed that the roof diaphragm sheathing delivers only a horizontal load to the sloping top of the shear wall. While there is a horizontal component of the diaphragm force, there is also a *vertical* component that must transfer into the wall somehow. This occurs because the overall force in the roof diaphragm must act in the plane of the diaphragm (by definition, a diaphragm carries forces only in its own plane). To see why this is true, see the free-body diagram in Figure 6 of a windward wall supported laterally at the top by sloping rafters. The wall-to-rafter connection is a pinned connection, so the rafter can only transmit an axial force to the wall. (This puts the wall studs in compression for the condition shown.) The axial force in the rafters transfers into the diaphragm as an in-plane force.

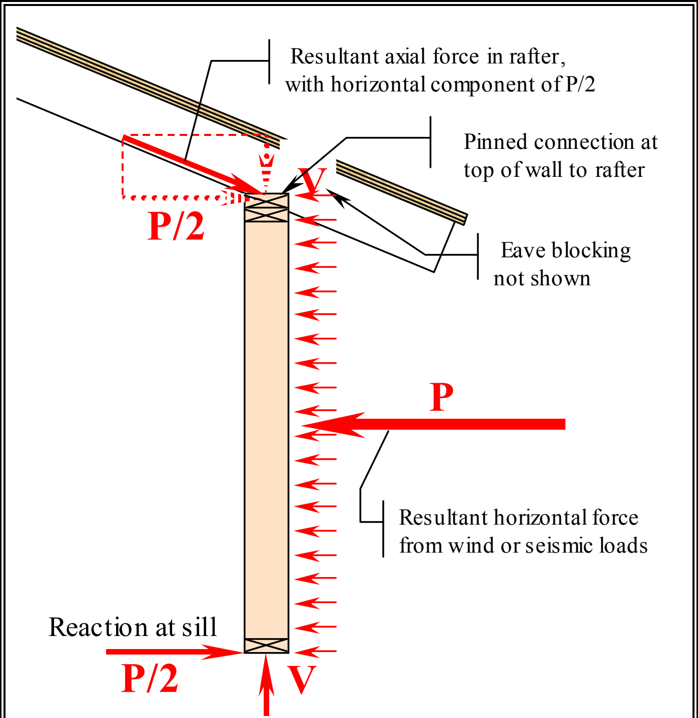

Figure 6 Free-body Diagram of wall. Pinned connections at top and bottom result in the rafter providing only an axial force at the top plate. This indicates that force transmitted to sheathing from rafter will be in the plane of the sheathing.

Let's approach the problem another way, and prove to ourselves that a diaphragm really can only carry forces within its own plane. Assume that the diaphragm does only deliver a horizontal force to the top of a sloping shear wall. The reaction of the wall on the diaphragm then has a component that acts perpendicular to the diaphragm's plane, and an in-plane component. The out-of-plane component could easily overstress the diaphragm sheathing in bending. Figure 7 shows a roof with a 7:12 (approximately 30 degree) slope. As an example, assume 3/8-inch sheathing with boundary nailing of 8-penny nails at 2 inch spacing. The APA tables incorporated into model codes list a maximum shear of 610 pounds per foot for this diaphragm (even though not too many engineers would specify such a diaphragm—but stick with me…).

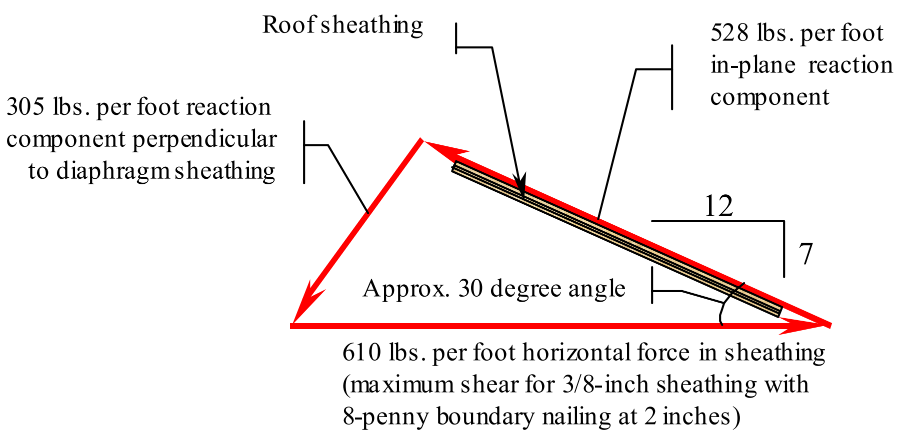

Figure 7 Force and reaction components acting on sloped diaphragm sheathing *if sheathing is assumed to transmit only a horizontal force.*
In this example the reaction component acting perpendicular to the sheathing induces a bending stress that exceeds the allowable stress by a factor of 14.
(Note that Word made the right-angle look like more than 90 degrees….)

For the out-of-plane reaction component, we get (610 lbs./ft.)(sin 30) = 305 lbs./ft.

If we have 24-inch rafter spacing (although few designers or builders would actually construct a roof like this, 3/8-inch sheathing is available with a 24-inch span rating), then the 305 pounds per foot out-of-plane force produces a bending moment of (24in.)(305 lbs.) = 7320 in.-lbs.

For a 3/8-inch thick solid board, the Section Modulus would be $\mathbf{bd^2/6}$, or $\mathbf{S=12in(0.375in)^2/6=0.28in.^3}$ Since the layers in plywood do not all have the same grain orientation, S is multiplied by a coefficient "K" to reduce it to a realistic value.

From the APA for 3/8 inch, Group I plywood, KS = 0.195 in.^{3} per foot. Solving for the bending stress, F_b=M/S

$$\mathbf{f_b = 7320\ in.\text{-}lbs./0.195\ in.^3 = 37{,}500\ psi.}$$

This would overstress *steel plate*!

For Group I structural panels, F_b = 2,000 psi (133%) = 2,670 psi, so:

$$\mathbf{f_b / F_b = 14}$$

Clearly this is an extreme example, using thin sheathing and assuming maximum allowable shear in the diaphragm. I use this to illustrate that the roof diaphragm cannot transfer a purely horizontal force to the sloping top chord of the shear wall.